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CHARACTER ISSUES

The Mathematical Romance: An Engineer's View of Mathematical Economics

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Abstract, Keywords, JEL Codes

I APPROACH MATHEMATICAL ECONOMICS FROM A SOMEWHAT unusual perspective. I have been fascinated with math since childhood and have had many opportunities to apply it in earning my Ph.D. in engineering, in my 30-year engineering career, and in teaching engineering to college students. I continue to enjoy math as recreation.

But my home study of Austrian economics, inspired by attendance at a Ludwig von Mises seminar in 1970, has made a skeptic of me. Now enrolled formally as a graduate student in economics, I approach the subject burdened neither by math phobia nor the zeal of a convert. I am simply curious: what is the appropriate role of mathematics in economics? Have economists misused math or been excessively preoccupied with the subject? Have they felt the romantic lure of the subject that I often felt as an engineer? Have they been sidetracked?

At first I thought economists might use math the way we engineers do, only more so. With less solid data to work from and less *paribus* in the *ceteris*, economists would be more circumspect in their use of math and more reliant on empirical observation or even “gut intuition.” Quite the

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opposite! Before offering my observations on mathematical economics I will review how engineers use math, with crash simulation offered as a modern example.

HOW ENGINEERS USE MATHEMATICS

My grandfather's "Kent's Mechanical Engineering Handbook," circa 1900, covered the entire field in one book, beginning with simple arithmetic. Most of the equations in the book are empirical and unit-specific. A century later, engineering remains an art as well as a science, but we expect much more mathematical sophistication of our students. "Walk the aisles of the university bookstore," says Deirdre McCloskey. "Open some of the advanced undergraduate books in physics (or in the much-despised civil engineering, for that matter). It makes the hair stand on end. Bessel functions abound. Group theory is routine" (McCloskey 2000, 215).

Bessel functions provide nice solutions to certain differential equations in cylindrical coordinates, but we don't actually use them much any more because modern computerized numerical methods can do the job much better. Real problems just don't provide the simple boundary conditions required of Bessel functions, Fourier series, or any other formulaic solutions. Nowadays we discretize the differential equations, i.e., convert them to algebraic equations which can be solved on a computer. Non-uniform boundary conditions are no longer a problem, but discretization does introduce approximations which must be managed.

The predominant method for solving boundary-value problems in civil and mechanical engineering is called finite element analysis. The method was conceived in 1943, but can only be applied using a computer and thus took off only in the 1970's. A "finite element model" is an abstract representation of a mechanical or structural system as an assembly of simple geometric shapes ("finite elements"). Illustration 1 shows such a model.[1]

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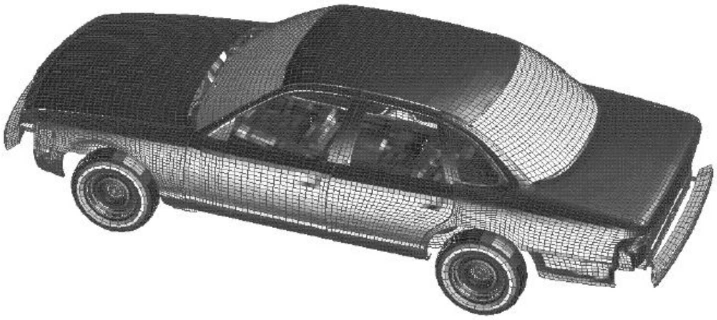
[1] All the figures shown are used by permission from S. W. Kirkpatrick, "Development and Validation of High Fidelity Vehicle Crash Simulation Models," SAE Technical Paper Series 2000-01-0627, © 2000 SAE International.

Illustration 1: Finite element model of a Ford Taurus



Each of the little quadrilateral areas shown in the illustration is a finite element. At each node where element boundaries intersect, six independent displacement variables are defined (three translations, three rotations), and an entire model may contain hundreds of thousands or even millions of variables. Forward integration in time is carried out using finite difference approximations to compute the response of the system to loads such as the impact forces in a crash simulation. An automobile is a challenging structure to model and the response to a crash impact is highly nonlinear, so successful modeling and numerical integration is a major challenge.

Illustration 2: Predicted deformation following a 35-MPH barrier impact

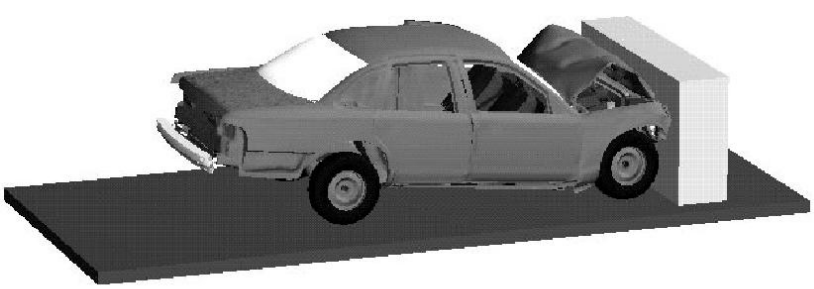
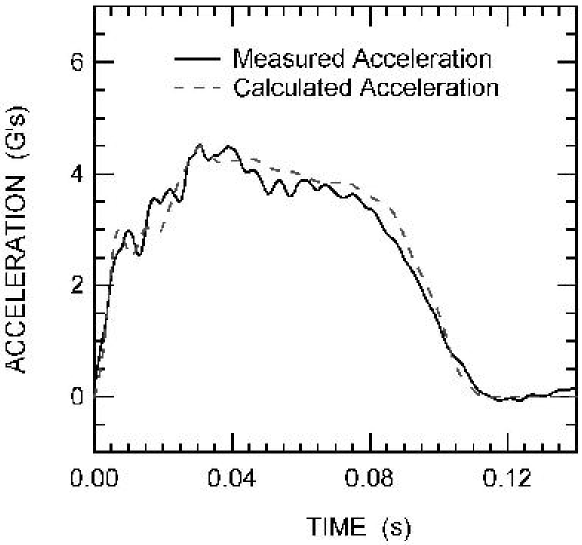


Illustration 2 shows the simulated deformed shape of the vehicle following a 35-mph impact with a standard barrier. Note that this is neither a photograph nor an artist's rendering, but rather a computer-generated image based on the results of the finite element analysis. It is reassuring that the picture looks realistic, but there are many approximations and pitfalls in this sort of analysis so it is important that computed results be validated by comparison with test data whenever possible. Illustration 3 shows such a comparison—a very close one in this case. (The car is of course decelerating. Depending on the orientation of the coordinate system, the values could be positive or negative, but are shown as positive for clarity.) The model has been used to predict the results of a crash test that was actually conducted in the lab. Computed accelerations on the bumper were compared to readings from an accelerometer mounted on the actual bumper. With the confidence that this comparison engenders, an engineer could try various design changes by modifying the model and re-running the simulation, saving the time and expense of a laboratory crash test for such design iterations. For example, one might add "crumple zones" designed to capture and dissipate kinetic energy. These would be "tested" in finite-element simulations, but prototype testing would still be done before any such addition were put into production.

Illustration 3: Comparison of post-crash bumper acceleration, measured and predicted



EXISTENCE, UNIQUENESS AND SUCH

When I was a graduate student and finite element analysis was new, there was a controversy about the interpolation functions that are the mathematical basis of finite elements. For the most common class of elements (thin shell elements), somebody proved that these function values and their first derivatives must be continuous across element boundaries in order to guarantee monotonic convergence of solutions as the mesh size is reduced (i.e., more and smaller elements). Such elements are called "conforming." One camp insisted that only conforming elements should be used. Others went ahead with mildly non-conforming elements because they demonstrated certain advantages that seemed worth the loss of conforming purity. In time the non-conforming elements proved themselves in practice and people just stopped worrying about the convergence theorem. In fact, there are a few rare pathological cases that finite elements fail to solve, but we live with this situation as physicists live with Heisenberg uncertainty, or number theorists with Gödel's theorem.

Engineers like to linearize equations whenever possible because linear equation solutions are so much simpler and cheaper than nonlinear. But car crashes are highly non-linear events and must be analyzed as such. With nonlinearity in the picture, the theorems that tell us how small the time step must be to assure convergence of the forward integration algorithm go out the window, as does any assurance of a unique solution. How can we continue in the face of such calamities? We just press on. If over time, certain practices lead to stable and verifiable results, we learn from experience and adopt those practices. Such attitudes give mathematicians heartburn, but engineers just shrug. Never mind the theorems, we have a job to do!

These are two examples of how engineers use (or misuse) math. How wrong I was in assuming I would find a similar approach in mathematical economics. I thought they too would leave the theorems and the existence proofs to the mathematicians and do everything possible to get to an answer, even cutting corners and relying on intuition as we engineers sometimes do, because there are such momentous problems awaiting their insights. But on the contrary, many of the economics papers I have looked at seem obsessed with math for its own sake, with real human problems hardly anywhere to be seen. The prevalence of existence proofs or statistical significance demonstrations without regard for what McCloskey calls "How Big?" was astonishing.

The predominance of abstract mathematical theory in economics was highlighted succinctly by Wassily Leontief in a letter to *Science* Magazine (Leontieff, 1982). Perusing the contents of the *American Economic Review*, he found that a slight majority of the articles presented mathematical models without any data, just 12% presented analysis without any math, while the rest were mainly empirical studies. "Page after page of economic journals are filled with mathematical formulas leading the reader from sets of more or less plausible but entirely arbitrary assumptions to precisely stated but irrelevant theoretical conclusions," he said. Math without data is unknown in engineering journals and rare in physics journals.

THE ECONOMICS OF MATHEMATICAL ENGINEERING

Engineers observe some rudimentary principles of economics, usually without knowing their names. As indicated above, there is specialization and a division of labor: proofs are left to the mathematicians and programming of analysis software to the programmers. Cost avoidance is another. Math is costly—car crash analyses, while generally much cheaper than actual crash tests, still require expensive talent and industrial-strength computers. Project managers try to minimize analysis costs and to calm the passions of analysts who may have fallen in love with their models and want to do "just one more run." And thirdly, projects have firm and objective goals. Even in government-funded engineering projects where market discipline is absent, physical discipline still rules. The Mars lander either works or it doesn't. Paradoxically, economists seem less attached to economic principles than engineers.

MATHEMATICAL ECONOMICS IN THE CLASSROOM

I am enrolled in a graduate microeconomics class which is almost entirely mathematical, though only at the level of elementary calculus. I am saddened by the experience. During the breaks, some students tell me it's just a hurdle for them to jump on their way to a degree. For them, perhaps the only harm done is the opportunity cost. What if, instead, they spent a semester applying Hazlitt's *Economics in One Lesson* to present-day problems? Surely they would come away better able to think critically about vital current issues. But what bothers me most is the prospect that one or two bright students may enter a Ph.D. program and vanish into a black hole of mathematical esoterica. At their age, I would have been entranced by the cool properties of the Cobb-Douglas production function. I would have heard the siren song loud and clear.

A glance at a contemporary econometrics textbook such as Peracchi's (2001) is instructive. The author says it is for advanced undergraduates or first-year graduate students. Nowhere is the term "econometrics" defined, nor is there any introduction that might have placed the subject matter in context. The book's very first sentence is "Consider a data matrix $\boldsymbol{z}$ of order $n\times q$, consisting of n observations on q variables that are numerical or can be represented as numerical." The topics include regression, sampling, time series, estimation, and related subjects, all good topics to be sure, useful in engineering, and likely in economics, as well. And there is nothing necessarily wrong with a pure math course in support of economics. Engineers take math courses, but all the math courses I ever took were given by the math department, and we all knew we were being prepared to apply the stuff, at which time theorems and derivations would be forgotten. But even though the math department was giving those courses, they took pains to include sample applications from physics or engineering. How strange, then, to find a text for use outside the math department that is pure mathematics. I found only one of about 400 end-of-chapter exercises in Peracchi that mentions human action, and that one begins, "In each period t, a farmer i combines his entrepreneurial ability Ai with labor input Lit in order to maximize expected profit $\Lambda_{it}=P_{it}Q_{it}-w_{it}L_{it}$" (Peracchi 2000, 422-423). It may be that any student who uses the Peracchi text in class has had the context and purpose of econometrics established by his instructors. Still, the nearly complete absence of any discussion of applications suggests that we are looking at math for the sake of math.

MODELING USED CAR PRICES

A recent paper (Stolyarev 2002) will serve to illustrate the puzzlement I feel as an engineer studying the application of mathematics to economics. The paper observes a bimodal distribution of used car sales with peaks at about 3-5 years and at about 10 years. Anyone who has owned a car could probably make some good guesses as to why. Mechanical problems start to surface after 3-5 years prompting many owners to sell. At ten years, most cars are just about finished. Most of us know that. But the paper claims to "explain" these phenomena using a mathematical equilibrium model. Without questioning the soundness of the mathematics employed, one might ask why this phenomenon needs explaining, or why common-sense explanations are inadequate, and most of all, just what has been gained in the end. The work described is in some ways similar to the task of building an engineering model and showing that it successfully explains some data gathered in the laboratory or in the field. One might admire the intelligence and the diligence and the cleverness that went into such a model. One might adopt it as a classroom example or apply it to subjects other than used cars. But the reason engineers undertake such an effort (other than in the classroom) is to improve designs or diagnose failures. No such prospect is offered in the Stolyarov paper. He claims his model "captures the observed resale patterns for autos." (Stolyarev 2002, 1391) By "capture" he of course means that his model successfully generates the observed bi-modal distribution. No mean feat, to be sure, yet one is left wondering just what has been gained by the capture. He does not claim that his explanation is superior to common-sense explanations nor that his model could be valuable to buyers or sellers of used cars. Perhaps economists who study the paper will develop a better "feel" for used-car markets (or for any market for used goods, as the model is more general than just used cars). I believe any lay person could read the Kirkpatrick crash-analysis paper and without understanding any of the math or engineering, get some idea of the practical value of the work. Can the same be said for the Stolyarov used car paper? Could his model provide some benefit to buyers or sellers of used cars, however indirectly?

MATHEMATICS VERSUS WISDOM

Mathematics can be very alluring. Professional mathematicians speak frequently of "beauty" and "elegance" in their work. Some say that the central mystery of our universe is its governance by universal mathematical laws. Practitioners of applied math likewise feel special satisfaction when a well-crafted simulation successfully predicts real-world physical behavior.

But while the mathematicians, some of them at least, are explicit about doing math for its own sake, engineers are hired to produce results and economists should be, too. It's fine if a few specialists labor at the outer mathematical edge of these fields, but the real needs and real satisfactions are to be found in applications.

Western civilization has brought us an explosion of human welfare: prosperity, longevity, education, the arts, and so on. We very much need the wisdom that economists can offer us to help understand and sustain this remarkable record. What good are engineers' accomplishments in crash simulations if the benefits are denied to the world by trade barriers, stifling regulation, congested highways, or bogus global warming restrictions? What can mathematical economics contribute to such vital issues? Not much, if Deirdre McCloskey is right when she says, "economics has learned practically nothing from the dual triumph of mathematical economics and econometrics." What if, as she says, "The best minds in economics have been diverted into an intellectual game, I say, with as much practical payoff as chess problems" (McCloskey 2000, 217). What if real answers to urgent problems could be delivered in plain English? Do economists have the courage to shun the romance of mathematics and produce such answers? Let us hope so.

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